

Program Transformation: Functional Programming Perspective.

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Abstract

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Program transformation is used in a wide range of applications including compiler constructions, optimization, program synthesis, refactoring, software renovation, among others. Complex program transformation are achieved though a number of consecutive modifications of a program, Transformations rules define basic modifications. A transformation strategy is an algorithm for choosing a path in a rewrite relation induced by a set of rules.

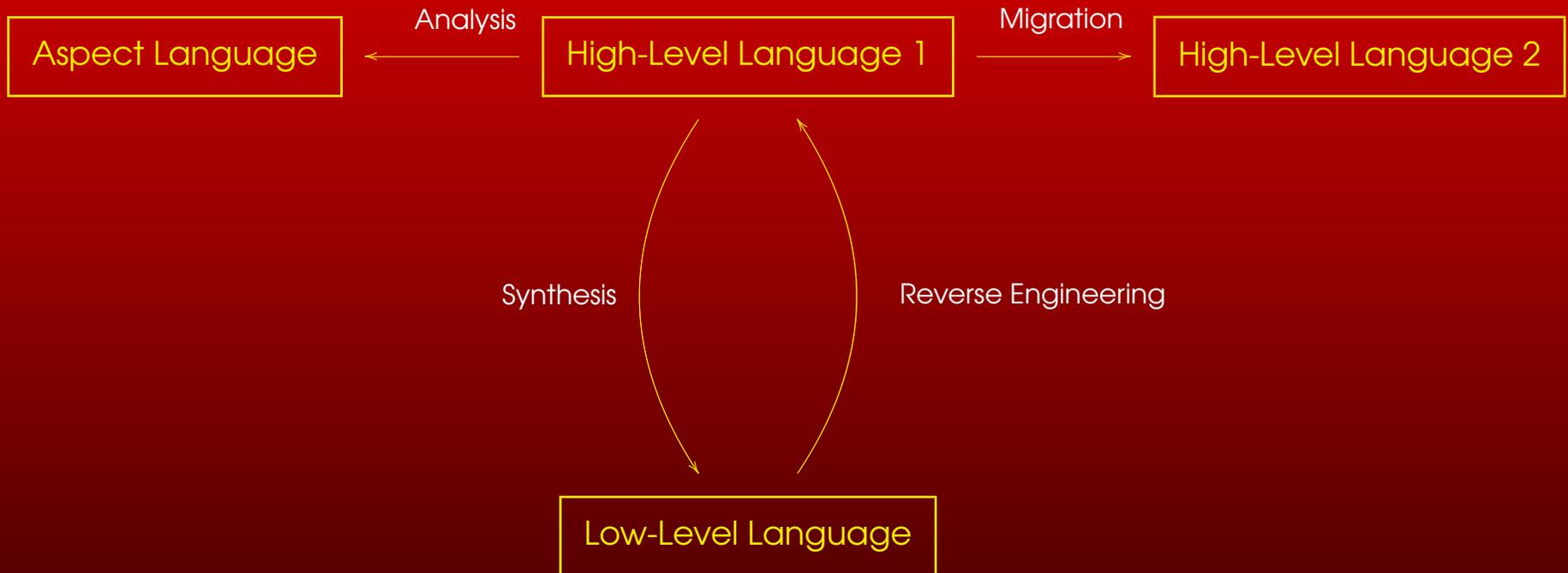
Program Transformation in a more general framework

Table 1: Taxonomy

Translation	Rephrasing
Migration Synthesis ○ Refinement ○ Compilation Reverse engineering ○ Decompilation ○ Architecture extraction ○ Documentation generation ○ Software visualization Analysis ○ Control-flow analysis ○ Data-flow analysis	Normalization ○ Simplification ○ Desugaring ○ Weaving Optimization ○ Specialization ○ Inlining ○ Fusion Refactoring ○ Design improvements ○ Obfuscation Renovation

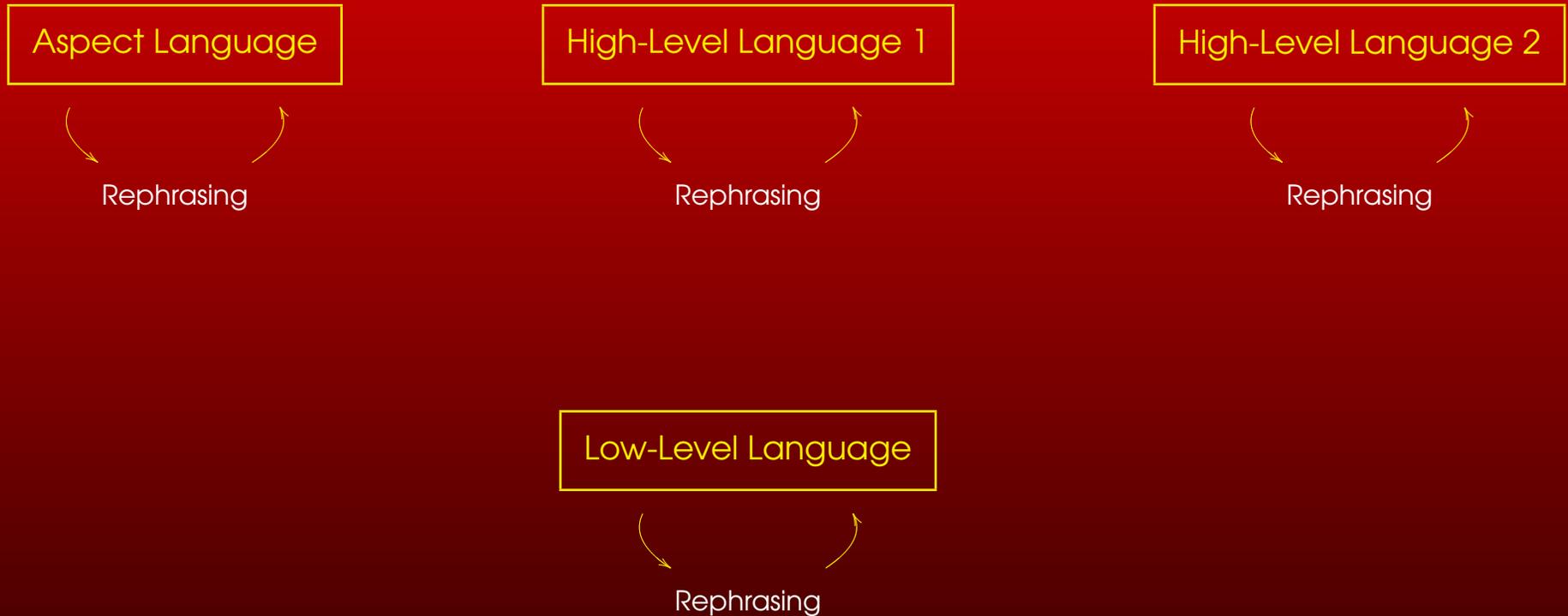
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Table 2: Translation



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Table 3: Rephrasing



Motivation.

Gérard Huet and **Bernand Lang** (1978) *“Proving and Applying Program Transformations Expressed with Second-Order Patterns”*

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“ There is a huge gap between software certification techniques and the theoretical tool defined for formal proofs of programs..”

Example 1.

Structural Recursion:

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fact(x) = if (x==0) return 1 else return x * fact (x-1)
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Iteration:

```
fact(x) = if (x==0) return 1
         else {
           result = x;
           x = x-1;
           While ( x !== 0) do {
             result = result * x;
             x = x-1};
           return result * 1 }
```

Example 2.

Structural Recursion:

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(rev x) = if (x== nil) return nil  
         else return (append (rev (tail(x)), [head(x)]))
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Iteration:

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(rev x) = if (x == nil) return nil
         else {
           result = [(head x)];
           x = (tail x);
           While ( x != nil) do {
             result = (append [(head x)] result);
             x = (tail x)};
           return (append result nil) }
```

Abstracting.

Σ :

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      else return h(d(x), f(c(x)))
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f(x) = if a(x) return b(x)
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        result = d(x);
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Instantiating patterns.

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f(x) = if a(x) return b(x)
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$$\sigma_1 \left\{ \begin{array}{l} f(x) \leftarrow \text{fact}(x); \\ a(x) \leftarrow (x == 0); \\ b(x) \leftarrow 1; \\ c(x) \leftarrow x - 1; \\ d(x) \leftarrow x; \\ h(u, v) \leftarrow u * v; \end{array} \right.$$

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- The Paradigm:
 1. recognize a pattern $(\Sigma, \Sigma', \mathcal{X})$
 2. validate such pattern.
 3. recognize whether or not a pattern is applicable to a given program.
 4. organize a system applying automatically those patterns.

Huet's Proposal.

Let $(\Sigma, \Sigma', \mathcal{X})$ be a transformation, and let $\mathcal{P}[t]$ be a program which contains t as subterm. We say the transformation is *applicable* in $\mathcal{P}[t]$ at t if and only if there exists a substitution σ for the free variable of Σ and Σ' such that:

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A problem occurs when for a certain context two different transformations are available, leading to two different programs, this is a well known problem for *Rewriting Systems*. In some cases it will be possible to prove that our set of transformations is *confluent*.

Glasgow Haskell ¹ Compiler: Compilation by transformation.

Compiler' Structure:

- The front end parses the source, does scope analysis and type inference, and translates the program into a small intermediate language called the *Core Language*.

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- The back end translates the resulting Core program into C, whence it is compiled to the machine code.

¹Haskell is a non-strict, typed, pure functional language

²Most of the "optimizations" appears at this level

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data Boolean = True | False
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data List a = Nil | Cons a (List a)
```

```
data Tree a = Leaf a | Branch (Tree a) (Tree a)
```

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“Desugarer”:

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“Desugarer”:

Haskell Code

\implies

Core Code

`f (sin x) y`

`let v = sin x in f v y`

`if C E1 E2`

`case C of {True -> E1; False -> E2}`

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A Core to Core Optimization:

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Deforestation/Fusion laws are program transformation by means of which intermediate data structures can be eliminated.

The interest in this particular technique is due to the fact that programs are often implemented in a compositional fashion, using intermediate data structures to connect such a components. This compositional style is suitable for modular programming, however may be inefficient both time and space.

Deforestation/fusion

From an example: "Sum of the square of n integers"

Version(1) Modular

```
SquareList :: [Int] -> [Int]
```

```
SquareList [] = 0
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SquareList (x :xs ) = [x2] ++ SquareList(xs)
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```

```
SumSqr :: [Int] -> Int
SumSqr = Sum ∘ SquareList
```

Deforestation/fusion

Version(2) UnModular

SumSquare :: [Int] -> Int

SumSquare [] = 0

SumSquare (x :xs) = x^2 + SumSquare(xs)

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A generalization?

Deforestation/fusion

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A generalization? Where could we get it?

Deforestation/fusion

Answer: **Category theory!!**, datatypes ³ as initial algebra for a given functor.

$$\begin{array}{ccc}
 \mathbb{T} & \xleftarrow{\alpha} & \mathbb{F}\mathbb{T} \\
 \downarrow ([f]) & \circlearrowleft & \downarrow \mathbb{F}([f]) \\
 \mathbb{A} & \xleftarrow{f} & \mathbb{A}
 \end{array}$$

³ $\alpha : \mathbb{F} \leftarrow \mathbb{F}\mathbb{T}$, is called the *initial algebra* for \mathbb{F} . Exists a unique $([f])$ called *catamorphism*

Deforestation/fusion

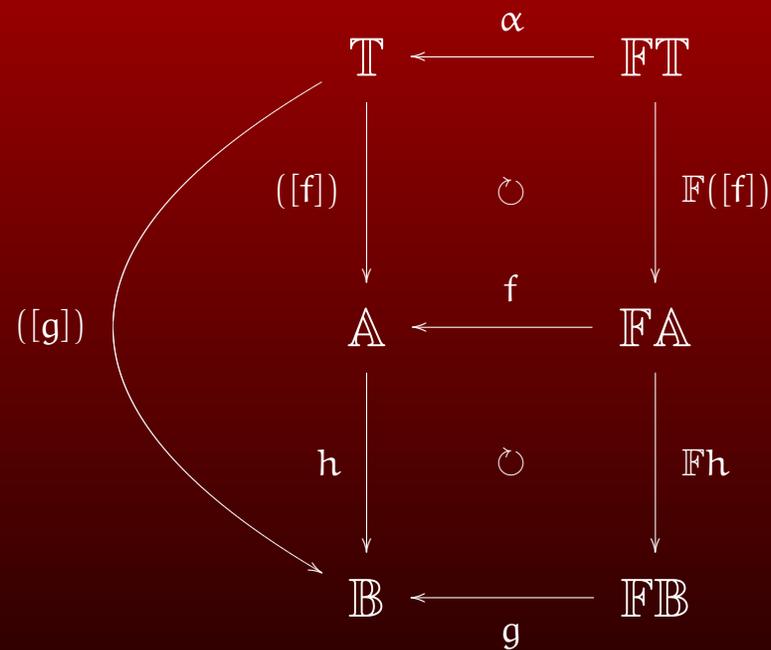
Fusion Law

- $([\alpha]) = \text{id}$ and $h \circ ([f]) = ([g]) \Leftarrow h \circ f = g \circ \mathbb{F}h$.

Deforestation/fusion

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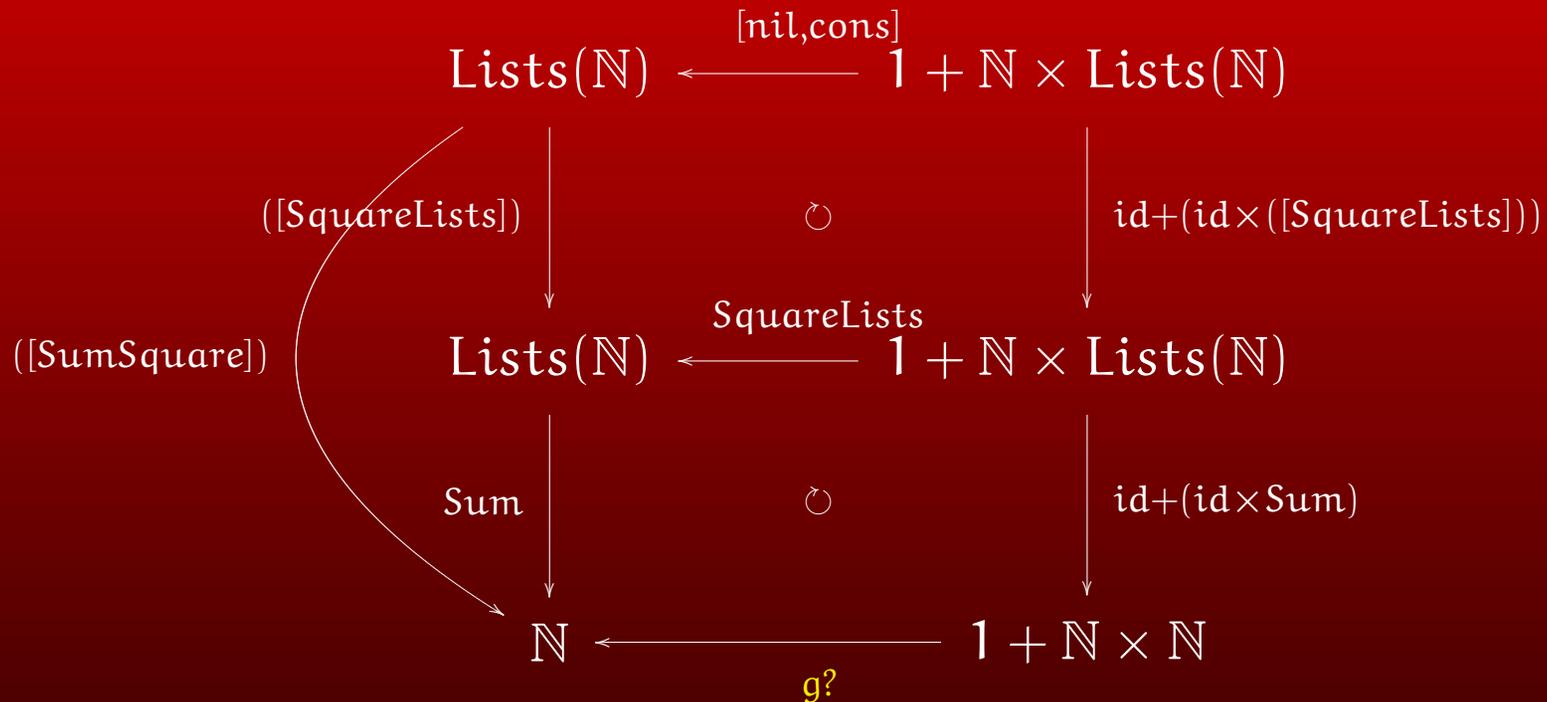


Deforestation/fusion

Back on our example: looking for g

Deforestation/fusion

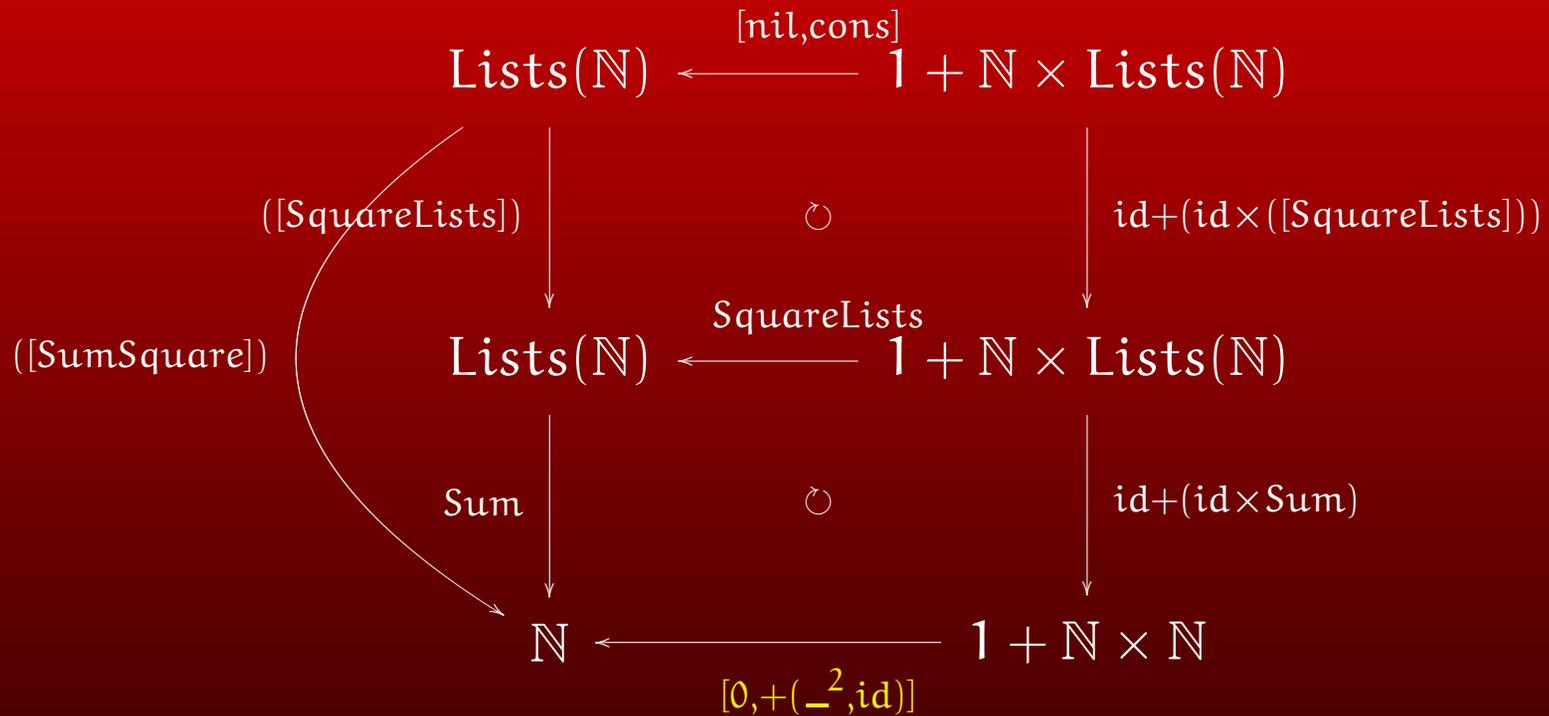
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Deforestation/fusion

Back on our example



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Deforestation/fusion

The key: observe that we found an appropriate $g = [0, +(-^2, \text{id})]$ such that the diagram commute i.e. we answer the question.

Is there some g such that $h \circ f = g \circ \mathbb{F}h$?

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Remark: this can be seen as a Matching problem!!

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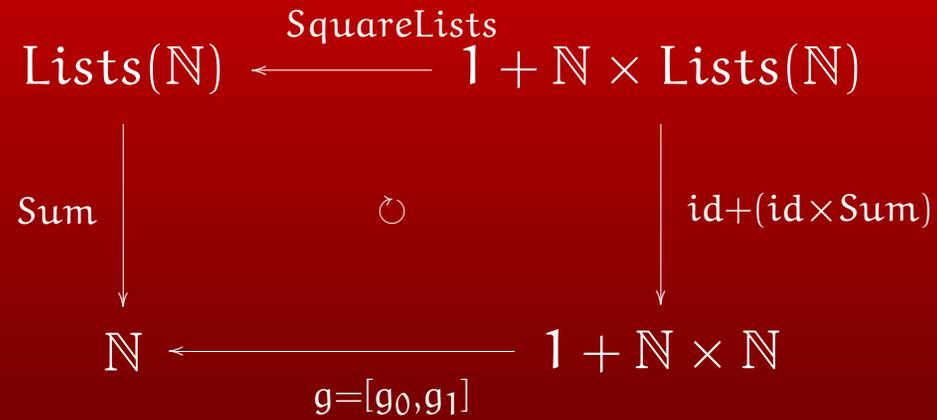
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Ref: **O. de Moor** and **G. Sittampalam**(1999) *Generic Program Transformation* (1999)

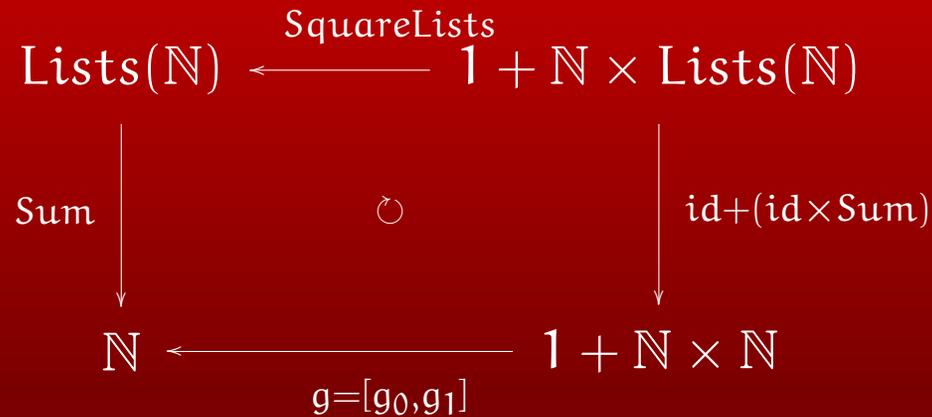
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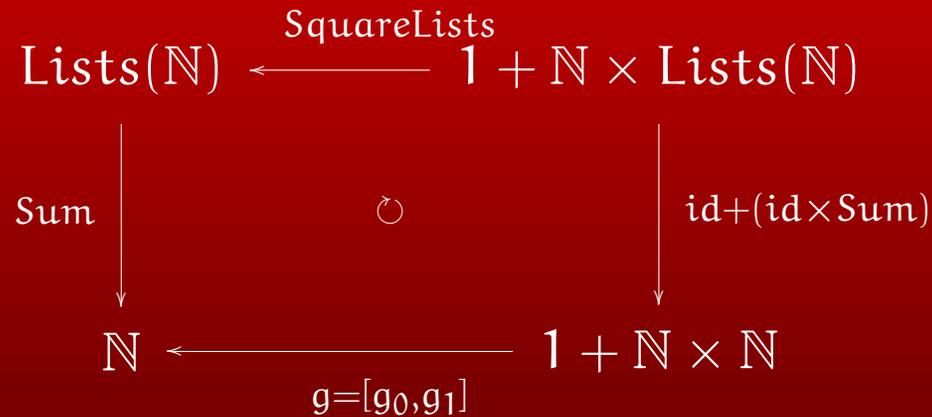


Translated to:

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Deforestation/fusion

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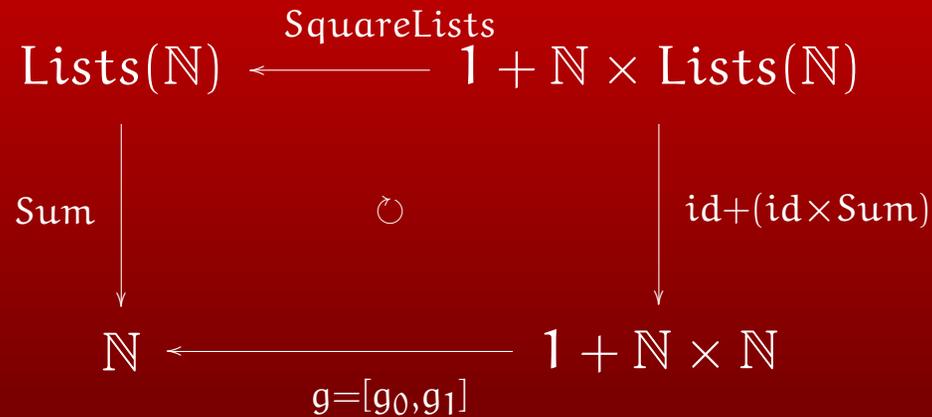
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`g` \implies `if (y = 1) g0 g1`

Higher Order Matching and HOAS

From this it's not hard to see that those programs can be encoded as a lambda terms (**HOAS**) i.e. the fusion law can be expressed as:

$$(t_{\text{Sum}} \ t_{\text{SquareLists}}) \stackrel{?}{=} (t_g \ [id, \langle id, t_{\text{Sum}} \rangle])$$

Higher Order Matching and HOAS

Results on Matching

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- More coming...

References

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